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On Mixed Boundary Value Problems for Sets of Partial Differential Equations with Constant Coefficients in Semi-Spaces

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Abstract—The solvability conditions of the over-determined boundary value problems for the set of PDE with constant coefficients in semi-spaces are obtained. The equations for complex amplitudes of the wave fields are considered as examples. The method of over-determined boundary value problem is used to obtain the boundary integral equation for the mixed boundary value problems.

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1. INTRODUCTION

General theory of partial differential equations with constant coefficients is based on the theory of distributions (for example, see [1], and [2]). If the boundary conditions with constant coefficients are imposed on the boundary of a semi-space, then Fourier integral transformation by the tangential variables enables us move to the boundary value problems for ordinary differential equations [3].

Essentially more complicated is the situation when on various parts of the boundary different boundary conditions are imposed (or when the coefficients in the boundary conditions are piecewise-constant). In this case, the standard method of investigation is integral equation method based on the potential theory [4]. The three-dimensional case is investigated in details in [5]. Some properties of boundary integral equations of mixed boundary value problems, including the statements of existence and uniqueness of their solutions, are studied in [6, 7].

In this work, we want to show that in order to study such mixed boundary value problems, method of over-determined boundary value problem (MODP) can be used. The main idea of the method is as follows [8, 9]. If we imposed on the boundary of the region more boundary conditions than it is necessary to determine a unique solution of the partial differential equation, then boundary functions cannot be chosen arbitrarily. The solvability conditions of the (auxiliary) over-determined problems together with mixed boundary conditions form a set of equations, which quite simply can be converted into integral equations on the part of the boundary. MODP is used also in reports [12–14] for solving some problems of diffraction of electromagnetic wave.

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